

The Thirty-Seventh Annual Eastern Shore High School Mathematics Competition

April 27, 2022

Individual Contest Exam

Instructions

There are twenty problems on this exam. Select the best answer for each problem.

Your score will be the number of *correct* answers that you select.

There is no penalty for incorrect answers.

The use of a calculator is **not** permitted on this exam.

No computational work is required for any of your multiple choice responses.

However, in the event of tie scores, after the multiple choice responses have been checked for problems 1-20, the responses and/or written computational work on the enclosed form for problems #18, #19 and #20 will then be used as tiebreakers.

1. Evaluate the following expression:

$$\log_4(\log_3(\log_2 8))$$

- a. 0 b. 1 c. 3 d. 4 e. 8

2. Suppose for two functions $f(x)$ and $g(x)$, $(f \circ g)(x) = |x| + 4$. The domain of $g(x)$ is the set of all real numbers. $f(x)$ and $g(x)$ could be which of the following?

- a. $f(x) = |x|$, $g(x) = x + 4$.
b. $f(x) = |x|$, $g(x) = x - 4$.
c. $f(x) = |x|$, $g(x) = -x + 4$.
d. a, b, and c are all correct.
e. a, b, and c are all incorrect.

3. Suppose $\sin(x) + 2 \sin(x) \cos(x) = 0$, where $0 \leq x \leq \pi$. Then,

- a. $x = 0$ b. $x = \pi$ c. $x = 0$ or $x = \pi$ d. $x = 0$, $x = \pi$, or $x = \frac{2\pi}{3}$ e. None of these

4. Alice and Bob each have a collection of dimes and quarters. Alice has exactly as many dimes as Bob has quarters, and she has exactly as many quarters as Bob has dimes. The combined value of Alice's collection is exactly twice as much as Bob's. What is the minimum number of coins Alice could have?

- a. 6 b. 9 c. 10 d. 14 e. 15

5. At a certain high school, some of the students in the marching band are also on the baseball team. Based on this, which of the following statements would imply that some marching band members are on the student council?

- a. All of the students on student council are on the baseball team.
b. Some students on the baseball team are on the student council.
c. Some students on the baseball team are not on the student council.
d. Statements a, b, and c.
e. None of the above.

6. Two circles have areas of 500 square units and 125 square units respectively. If the larger circle is drawn first, then which of the following is true for the radius scale factor, S , that should be used to draw the smaller circle?

- a. $0.1 \leq S < 0.2$ b. $0.2 \leq S < 0.3$ c. $0.3 \leq S < 0.4$ d. $0.4 \leq S < 0.5$ e. $0.5 \leq S < 1$

7. Let a_n denote the sum of the first n terms of the geometric series $1 + 2022 + 2022^2 + 2022^3 + \dots + 2022^n$, starting with $a_0 = 1$. So, $a_0 = 1$, $a_1 = 1 + 2022 = 2023$, $a_2 = 1 + 2022 + 2022^2 = 4090507$, etc.

Which of the following is the value of $\frac{a_9}{a_4}$?

- a. 2021
b. 2021^5
c. $2022^5 - 1$
d. $2022^5 + 1$
e. $2002^{10} - 2022^5$

8. Suppose a and b are integers greater than 3 such that $ab = 2022$. What is $a + b$?

- a. 343 b. 345 c. 677 d. 1013 e. 2010

9. If $f(x) = (x - 3)(x - k)$ and if the remainder is 16 when $f(x)$ is divided by $(x + 1)$, then what is k ?

- a. -2 b. 0 c. 2 d. 3 e. 4

10. If $x^2 - 5x + 6 = 0$, what does $x^4 - 10x^3 + 26x^2 - 5x + 2$ equal?

- a. 2 b. 3 c. 24 d. 32 e. 36

11. Suppose a, b are nonzero real numbers and $a < b$. Which of the following statements are always true?

- a. $a^2 < b^2$ b. $a^3 < b^3$ c. $\frac{1}{a} > \frac{1}{b}$ d. Both b and c e. a, b, and c

12. What interval contains the solutions to the equation $\sqrt{2x + 1} = 1 + \sqrt{x}$?

- a. [-2, 3] b. [0, 5] c. [2, 7] d. [4, 9] e. [6, 11]

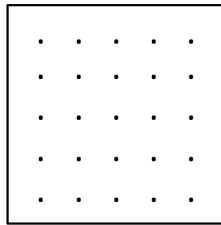
13. Find the root of the equation $2 + \log(\sqrt{1 + x}) + 3 \log(\sqrt{1 - x}) = \log(\sqrt{1 - x^2})$.

- a. $\frac{9}{10}$ b. $\frac{99}{100}$ c. $\frac{1}{9}$ d. $\log\left(\frac{9}{10}\right)$ e. $\log(2)$

14. If $x > 0$, $x^2 = 2^{64}$ and $x^x = 2^y$, then what is the value of y ?

- a. 2 b. 2^{11} c. 2^{32} d. 2^{37} e. 2^{64}

15. Consider a 5×5 rectangular grid (as shown below) where the distance between any two consecutive horizontal or vertical lattice points on the grid is one unit. If two of these points are used as endpoints of a segment, segments of various lengths can be drawn. If only one segment of each possible length is drawn on the grid, what is the sum of the lengths of segments with integral lengths?



- a. 3 units b. 6 units c. 10 units d. 15 units e. 21 units

16. The respective coordinates of points A, B, C , and D are $(0, 0), (0, 10), (100, 15)$ and $(100, 0)$. Segments AC and BD intersect at point E . Determine the area of triangle AED .

- a. 60 square units
b. 120 square units
c. 180 square units
d. 240 square units
e. 300 square units

17. A cat toy consists of 3 detachable balls: one catnip, one silvervine, and one gall fruit. The toy's description says that 70% of cats like catnip, 80% of cats like silvervine, and 90% of cats like gall fruit. What is the approximate chance that a cat will be interested in the toy (i.e., be attracted by any of the three balls?) You may assume that the following are independent events: a cat is attracted to catnip, a cat is attracted to silvervine, a cat is attracted to gall fruit.
- a. 0.6% b. 5.5% c. 80% d. 99.4% e. None of these

18. Consider the following sequence:

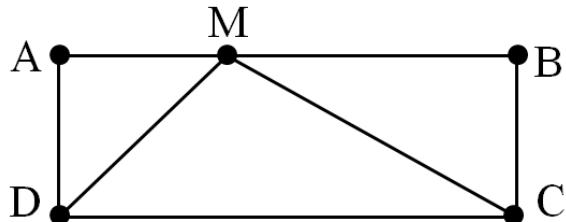
$$5, 8, 7, 5, 9, 2, 11, \dots$$

Let S_n denote the sum of the first n terms of the sequence. What is the smallest value of n for which $S_n < 0$?

- a. $n = 27$
 b. $n = 28$
 c. $n = 54$
 d. $n = 55$
 e. $n = 56$
19. Evaluate the following expression:

$$\sqrt{29 - 12\sqrt{5}} - \sqrt{29 + 12\sqrt{5}}$$

- a. -12 b. -6 c. 6 d. 12 e. 36
20. In the rectangle shown below, $AM = \frac{s}{t}AB$, where s and t are positive integers with $s < t$.



If the area of $\triangle ADM$ is 1, what is the area of $\triangle BCM$?

- a. $\frac{s}{t}$ b. $\frac{t}{s}$ c. $\frac{t}{2s}$ d. $\frac{t-s}{t}$ e. $\frac{t-s}{s}$