The Thirty-Fourth Annual
Eastern Shore High School Mathematics Competition

November 9, 2017

Team Contest Exam

Instructions

Answer as many questions as possible in the time provided. To receive full credit for a correct solution, show all work and provide a clearly written explanation. Solutions will be judged based on correctness, completeness and clarity. (Little credit, if any, will be given for a solution consisting of just a number or a single sentence.) Calculators are allowed **only** on the team contest exam.

All work and answers must be written on the provided sheets of plain white paper. Use only one side of each sheet of paper, and start each new problem on a new sheet of paper. Write your team name (that is, the name of the school which you are representing) at the top of each sheet that you turn in for scoring.

At the start of the team round, your team will receive a copy of only Problem 1. Your team must submit a response to Problem 1 within the first 15 minutes of the team round time interval.

When you submit your response for Problem 1, you will receive a copy of Problem 2 and a copy of Problem 3. Your team will then have the time remaining in the team round to complete a response for each problem.

Note: if your team completes Problem 1 before the end of the allotted time, you may submit it and receive copies of Problem 2 and Problem 3 in advance.

1. "ABC", "DEF" and "GHI" are distinct 3-digit numbers. Furthermore, the nine digits are non-zero and distinct.

   i.e., $\{A, B, C, D, E, F, G, H, I\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

   Part 1: Consider the sum: $ABC + DEF = GHI$.

   If $GHI = 981$, determine all possible distinct solutions to the problem $ABC + DEF = 981$.

   i.e., $\{A, B, C, D, E, F\} = \{2, 3, 4, 5, 6, 7\}$ and $\{G, H, I\} = \{1, 8, 9\}$.
   Note: $ABC + DEF = 981$ and $DEF + ABC = 981$ are not considered to be different solutions.

   Enter your solutions in the table shown below. List the smaller addend first in each of your distinct solutions. You may not need all of the cells shown to list all of the distinct solutions.

ABC	+	*DEF*	=	*GHI*		*ABC*	+	*DEF*	=	*GHI*		*ABC*	+	*DEF*	=	*GHI*
	+		=	*981*			+		=	*981*			+		=	*981*
	+		=	*981*			+		=	*981*			+		=	*981*
	+		=	*981*			+		=	*981*			+		=	*981*
	+		=	*981*			+		=	*981*			+		=	*981*

   Part 2: Consider the sum: $ABC + DEF = GHI$.

   If the digits G, H and I are distinct and are elements of the set $1, 8, 9$ and "GHI" $\neq 981$, determine all possible distinct solutions to the problem $ABC + DEF = GHI$.

   i.e., $\{G, H, I\} = \{1, 8, 9\}$ and $\{A, B, C, D, E, F\} = \{2, 3, 4, 5, 6, 7\}$.
   Note: ABC + DEF = GHI and DEF + ABC = GHI are not considered to be distinct solutions.

   Enter your solutions in the table shown below. List the smaller addend first in each of your distinct solutions. You may not need all of the cells shown.

ABC	+	*DEF*	=	*GHI*		*ABC*	+	*DEF*	=	*GHI*		*ABC*	+	*DEF*	=	*GHI*
	+		=				+		=				+		=	
	+		=				+		=				+		=	
	+		=				+		=				+		=	
	+		=				+		=				+		=	
	+		=				+		=				+		=	
	+		=				+		=				+		=	
	+		=				+		=				+		=	
	+		=				+		=				+		=	

2. "ABC", "DEF" and "GHI" are distinct 3-digit numbers. Furthermore, the nine digits are non-zero and distinct.

   i.e., $\{A, B, C, D, E, F, G, H, I\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

   Consider the sum $ABC + DEF = GHI$.

   (a) Explain why GHI cannot equal 345.

   (b) Explain why GHI cannot equal 548.

3. The digits 1,2,3, and 4 can be arranged in a multiplication equation where each digit is used exactly once: $3 \times 4 = 12$.

   (a) Find a whole-number multiplication equation using the digits 1, 2, 3, 4, and 5, where each digit is used exactly once.

   (b) Find a whole-number multiplication equation using the digits 1, 2, 3, 4, 5, and 6, where each digit is used exactly once.