The Thirty-Third Annual
Eastern Shore High School Mathematics Competition

November 10, 2016

Team Contest Exam

Instructions

Answer as many questions as possible in the time provided. To receive full credit for a correct solution, show all work and provide a clearly written explanation. Solutions will be judged based on correctness, completeness and clarity. (Little credit, if any, will be given for a solution consisting of just a number or a single sentence.) Calculators are allowed **only** on the team contest exam.

All work and answers must be written on the provided sheets of plain white paper. Use only one side of each sheet of paper, and start each new problem on a new sheet of paper. Write your team name (that is, the name of the school which you are representing) at the top of each sheet that you turn in for scoring.

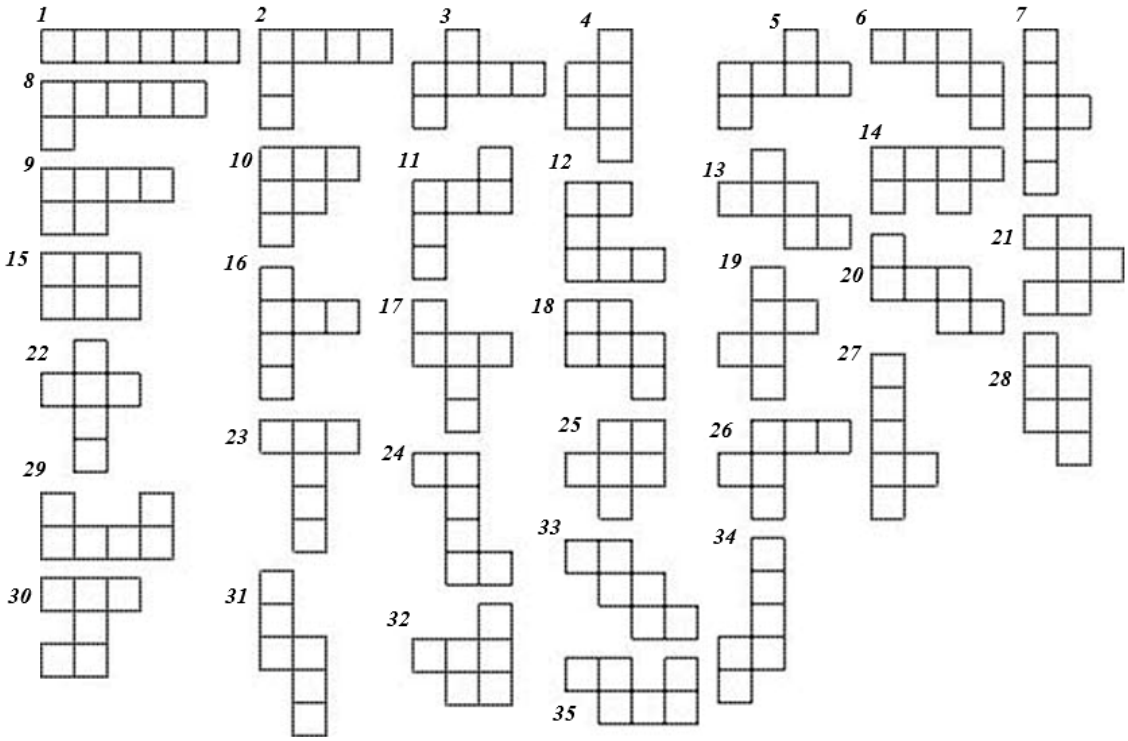
At the start of the team round, your team will receive a copy of only Problem 1. Your team must submit a response to Problem 1 within the first 15 minutes of the team round time interval.

When you submit your response for Problem 1, you will receive a copy of Problem 2 and a copy of Problem 3. Your team will then have the time remaining in the team round to complete a response for each problem.

Note: if your team completes Problem 1 before the end of the allotted time, you may submit it and receive copies of Problem 2 and Problem 3 in advance.

PLEASE NOTE THAT QUESTION 1 IS PRINTED ON BOTH SIDES OF THIS PAGE.

1. **Part 1**: A polyomino is a planar geometric figure formed by joining one or more congruent squares edge to edge. A hexomino is a polyomino composed of six squares, connected orthogonally. If translations, rotations and reflections are allowed, there are 35 different hexominoes (referred to as "free hexominoes").



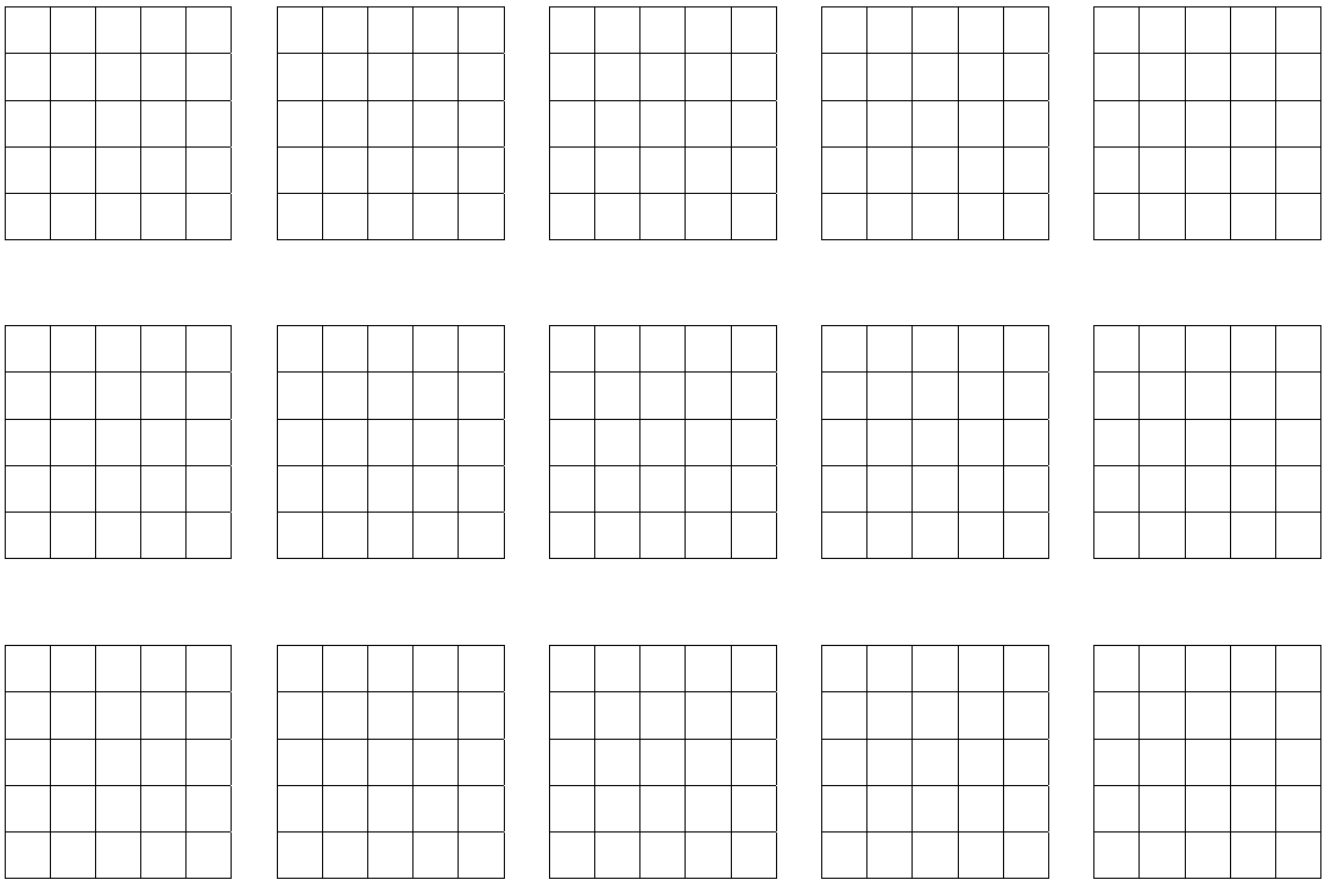
Encircle all of the respective numbers (in the table shown below) that identify free hexominoes that can be folded into a cube.

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35

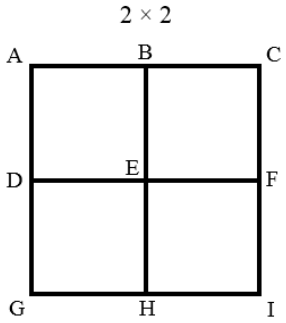
Note: credit will be deducted from your score for each incorrect entry.

Part 2: A pentomino is a polyomino composed of five congruent squares, connected orthogonally. If translations, rotations and reflections are allowed, the set of different pentominoes are called free pentominoes. On the grids provided below sketch (by shading the appropriate squares) the complete set of free pentominoes. Sketch only one free pentomino per grid. All of the grids **MAY NOT** be needed.

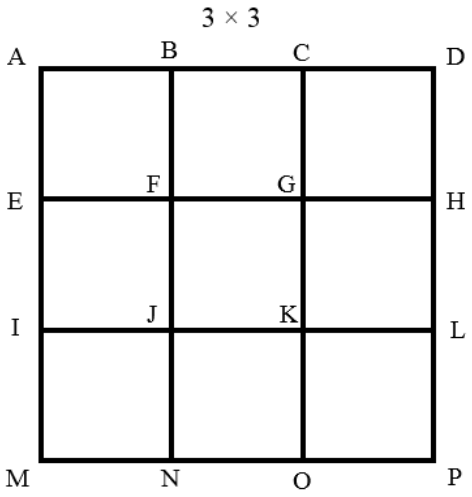
Note: credit will be deducted from your score for each "free pentomino" sketched which is equivalent to another.



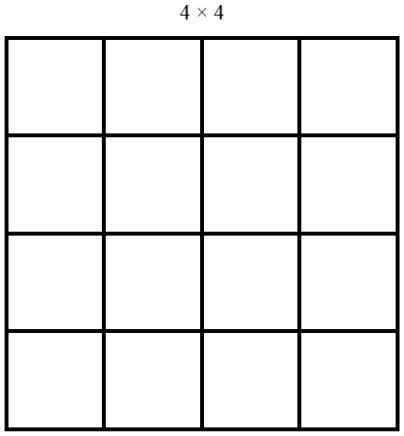
2. a. A number of squares are made by the grid lines in the following diagram.
List all of the squares you find in "Alphabetical Vertex Order," with the vertices in alphabetical order. For example "Alphabetical Vertex Order" for the square with vertices A, C, I and G is "ACGI." Answers that are not in "Alphabetical Vertex Order" will not be considered.

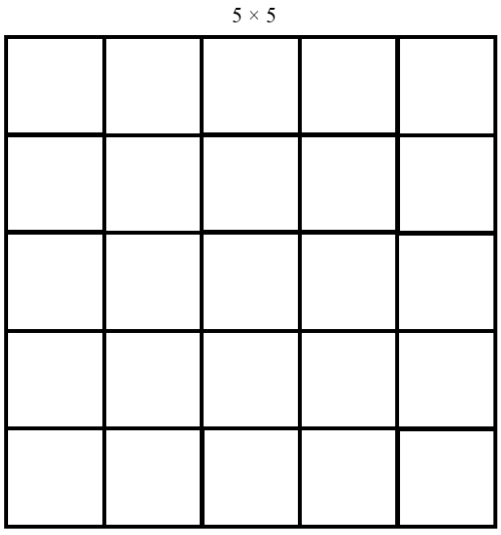


b. A number of squares are made by the grid lines in the following diagram.
List all of the squares you find in "Alphabetical Vertex Order," with the vertices in alphabetical order. Answers that are not in "Alphabetical Vertex Order" will not be considered.



c. Find a formula for the total number of squares in an $n \times n$ grid where n is any natural number larger than 1. You may use the grids below to help but your answer must be for a general $n \times n$ grid.





3. Nine squares with side lengths 1, 4, 7, 8, 9, 10, 14, 15 and 18 (measured in the same units) can be arranged to form a rectangle. After the nine squares are arranged to form the rectangle there are no gaps between any squares and there are no overlapping squares.

   What are the dimensions of the rectangle? Provide appropriate justification for your solution.