

The Thirty-First Annual Eastern Shore High School Mathematics Competition

November 6, 2014

Team Contest Exam

Instructions

Answer as many questions as possible in the time provided. To receive full credit for a correct solution, show all work and provide a clearly written explanation. Solutions will be judged based on correctness, completeness and clarity. (Little credit, if any, will be given for a solution consisting of just a number or a single sentence.) Calculators are allowed **only** on the team contest exam.

All work and answers must be written on the provided sheets of plain white paper. Use only one side of each sheet of paper, and start each new problem on a new sheet of paper. Write your team name (that is, the name of the school which you are representing) at the top of each sheet that you turn in for scoring.

At the start of the team round, your team will receive a copy of only Problem 1. Your team must submit a response to Problem 1 within the first 15 minutes of the team round time interval.

When you submit your response for Problem 1, you will receive a copy of Problem 2 and a copy of Problem 3. Your team will then have the time remaining in the team round to complete a response for each problem.

Note: if your team completes Problem 1 before the end of the allotted time, you may submit it and receive copies of Problem 2 and Problem 3 in advance.

1. One hundred twenty-five congruent cubes have been used to form a “big cube” that is solid. You drill the center of each corner cube of the side that faces you and continue to drill all the way through to the other side of the “big cube.” You repeat the process starting on the top side of the “big cube.” After you have finished drilling the top you examine the 125 cubes that formed the “big cube,” how many of those cubes do not have a whole drilled through them? Provide appropriate justification for your answer.

Note: you have been given 30 cubes to possibly assist you.

2. a. Provide justification that 541 is a prime number.

b. Find a prime number larger than 541. Provide justification that the number is a prime number.

3. A **palindrome** is a number that is the same when written forwards or backwards. For example, 12321 is a palindrome.

Find two 3-digit palindromes that are also prime numbers and differ by 10. Provide justification that each is a prime number.