

# The Thirty-First Annual Eastern Shore High School Mathematics Competition

November 6, 2014

## Individual Contest Exam

### Instructions

There are twenty problems on this exam. Select the best answer for each problem.

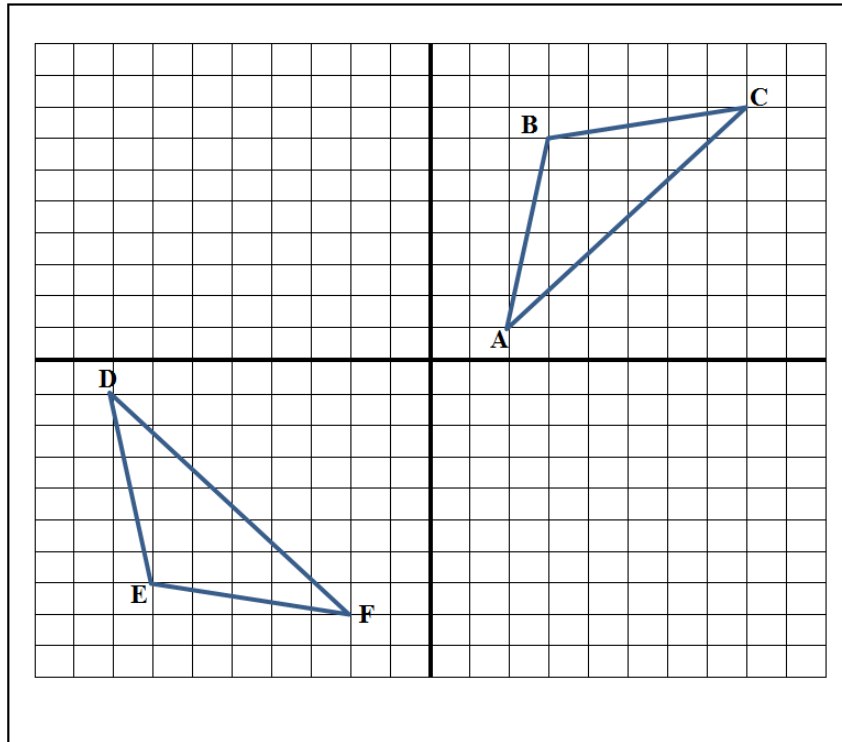
Your score will be the number of *correct* answers that you select.

**There is no penalty for incorrect answers.**

The use of a calculator is **not** permitted on this exam.

In the event of tie scores, #18, #19 and #20 will be used as tiebreakers.

1. Consider  $\triangle ABC$  and  $\triangle DEF$  that are shown on the grid. Which of the following is true?



- a.  $\triangle DEF$  is a reflection of  $\triangle ABC$  in the origin.
  - b.  $\triangle DEF$  is a rotation and reflection of  $\triangle ABC$ .
  - c.  $\triangle DEF$  is a glide rotation of  $\triangle ABC$ .
  - d.  $\triangle DEF$  is a glide reflection of  $\triangle ABC$ .
  - e.  $\triangle DEF$  is a glide of  $\triangle ABC$ .
2. You are enjoying a circular “Lazy River” amusement ride at a popular water park. The total number of people on the ride is  $\frac{1}{7}$  of the number of people in front of you, plus  $\frac{7}{8}$  of the number of people behind you. If  $K$  represents the total number of people on the ride, then
- a.  $40 < K < 80$
  - b.  $90 < K < 130$
  - c.  $140 < K < 180$
  - d.  $190 < K < 230$
  - e. cannot be determined
3. Suppose you received a mega-sized chocolate bar for your birthday. You love chocolate, so you want to make the bar last as long as possible. You decide to eat half of what is left of the bar each day. After how many days will you have less than  $\frac{1}{20}$  of the chocolate bar left?
- a. 2
  - b. 3
  - c. 4
  - d. 5
  - e. 6

4. For which real values of  $x$  is  $\ln\left(\frac{x^2 - 1}{x + 1}\right) = 0$ ?
- a.  $x = 1$       b.  $x = -1, 1$       c.  $x = -1, 2$       d.  $x = 2$       e. none

5. Evaluate  $\sin 75^\circ \cdot \sin 45^\circ$ .

- a.  $90^\circ$       b.  $0$       c.  $\frac{1}{4}$       d.  $\frac{1 + \sqrt{3}}{4}$       e.  $\frac{\sqrt{3}}{2}$

6. Below is the table describing books on Brian's book shelf:

	Fiction	Non-fiction	Textbooks	Row total
Hard cover	8	33	21	62
Soft cover	12	28	15	55
Column total	20	61	36	117

A random book is selected from Brian's bookshelf. Arrange the following probabilities from smallest to largest:

- I. P(a fiction book)  
 II. P(a hard cover and a non-fiction book)  
 III. P(a textbook, given it is a hard cover book)  
 IV. P(a hard cover, given it is a non-fiction book)

- a. I, II, III, IV  
 b. I, III, II, IV  
 c. I, III, IV, II  
 d. I, II, IV, III  
 e. None of these

7. Consider the following statement: If a mub is extrinsic, then it is concentrically sabid.

Mark thinks the above statement is false and wants to construct a counterexample to support his thinking. What must Mark do?

- a. He must find a mub that is neither concentrically sabid nor extrinsic.  
 b. He must find a mub that is not extrinsic.  
 c. He must find a mub that is concentrically sabid but not extrinsic.  
 d. Any of the choices a, b, or c will provide a counterexample.  
 e. None of the above choices is correct.

8. How must the numbers  $a$  and  $b$  be related so that they always satisfy the incorrect identity  $\ln(a + b) = \ln(a) + \ln(b)$ ?

- a.  $a(b - 3) = (a - 1)b$   
 b.  $a^2 = 2b$   
 c.  $a(b - 1) = b$   
 d.  $a^2 + a = b - 1$   
 e.  $a^2b^2 = (a - 1)(b - 1)$

9. It is known that the proportion of observations that lie within 2 standard deviations around the mean is at least  $\frac{3}{4}$  or 75%. It is also the case that at least  $\frac{8}{9}$  (89%) of observations lie within 3 standard deviations around the mean. A manufacturer of car batteries reports that the average (mean) lifetime of the company's batteries is 48 months. It is also reported that the standard deviation is 2 months.

Consider the statements below:

- I. At least 75% of the batteries should last between 46 and 50 months.
- II. At least 89% of the batteries should last between 42 and 54 months.
- III. At most 25% of the batteries should last longer than 52 months.
- IV. At most 11% of the batteries should last fewer than 42 months.

Which statement(s) above is/are true?

- a. Only I
  - b. Only II
  - c. II and III only
  - d. II and IV only
  - e. II, III, and IV
10. A student's grade in an introductory statistics course is determined by grades on quizzes, homework, tests, and a final exam. The professor counts the average of 3 tests as 40%, quizzes as 30%, homework as 10%, and the final exam as 20% of the final grade. Samantha has a test average of 88, a quiz average of 89, and homework average of 95. What is the minimum grade that Samantha needs to earn on the final exam to earn an A (90% or better) in the course?
- a. 90.67
  - b. 88
  - c. 93
  - d. 95.4
  - e. not enough information provided
11. Triangle  $ABC$  has side lengths  $AB = 5$ ,  $BC = 3$ , and  $m\angle ABC = 120^\circ$ . Find the length of side  $AC$ .
- a.  $\sqrt{34}$
  - b. 6
  - c.  $3\sqrt{5}$
  - d.  $4\sqrt{3}$
  - e. 7
12. On a standard clock face, what is the measure of the angle formed by the hour hand and the minute hand at 10:40?
- a.  $50^\circ$
  - b.  $60^\circ$
  - c.  $70^\circ$
  - d.  $80^\circ$
  - e.  $90^\circ$
13. Suppose the following argument is valid:
- Premise: All greens are loud.  
Premise: No blues are loud.  
Premise: [see below]
- Conclusion: No greens are circles.
- What is the third premise? (Note: only one of the following choices results in a valid argument!)
- a. All circles are blues.
  - b. All blues are circles.
  - c. No circles are blues.
  - d. No blues are squares.
  - e. No blues are greens.
14. Suppose  $a$  and  $b$  are real numbers. The curves  $x = y^2 - a$  and  $x = -y^2 - b$  are guaranteed to have two points of intersection when
- a.  $a = b$
  - b.  $a > 0$
  - c.  $b > 0$
  - d.  $a > b$
  - e.  $b > a$

15. If  $f(x) = \frac{x}{|x|}$  and  $g(x) = \frac{x}{|x|} + \frac{|x|}{x}$ , then  $(g \circ f)(x) =$
- a. 2      b.  $2x$       c.  $\frac{1}{2} \left( \frac{x}{|x|} + \frac{|x|}{x} \right)$       d.  $f(x)$       e.  $g(x)$
16. There are 10 numbers in a sequence and each is 2 more than the previous number. If the 5th number is 30, what is the sum of the sequence?
- a. 270      b. 300      c. 308      d. 310      e. 312
17. In a new smartphone wallpaper app, the user can fill the screen with their favorite color. The app starts when the phone is not being used and covers up half of the remaining screen each second. The phone is set down on the desk, and after exactly  $t$  seconds have elapsed, the user looks at the phone.  $\frac{511}{512}$  of the screen is filled. Find  $t$ .
- a. 5 seconds  
b. 7 seconds  
c. 8 seconds  
d. 9 seconds  
e. 10 seconds
18. The number 2014 is one more than a multiple of 11, ( $2014 = 11 \times 183 + 1$ ), and it is one less than a multiple of 13 ( $2014 = 13 \times 155 - 1$ ). Let  $y$  be the next larger whole number that is both one more than a multiple of 11 and one less than a multiple of 13. What is true about  $y$ ?
- a. It is between 2015 and 2050  
b. It is between 2051 and 2100  
c. It is between 2101 and 2150  
d. It is between 2151 and 2200  
e. It is greater than 2200
19. How many distinct positive divisors (or factors) does 2014 have?
- a. 4      b. 6      c. 8      d. 10      e. 12
20. The year 2014 has the following property: the number in its thousands place (2), multiplied by the sum of its digits (7), gives us the number formed by its last two digits (14).
- When will be the next time that a year has this property again?
- a. Between 2015 and 2100  
b. Between 2101 and 2200  
c. Between 2201 and 2300  
d. Sometime after 2301  
e. It will never happen again.