The Thirtieth Annual
Eastern Shore High School Mathematics Competition

November 7, 2013

Team Contest Exam

Instructions

Answer as many questions as possible in the time provided. To receive full credit for a correct solution, show all work and provide a clearly written explanation. Solutions will be judged based on correctness, completeness and clarity. (Little credit, if any, will be given for a solution consisting of just a number or a single sentence.)

All work and answers must be written on the provided sheets of plain white paper. Use only one side of each sheet of paper, and start each new problem on a new sheet of paper. Write your team name (that is, the name of the school which you are representing) at the top of each sheet that you turn in for scoring.

At the start of the team round, your team will receive a copy of only Problem 1. Your team must submit a response to Problem 1 within the first 15 minutes of the team round time interval.

When you submit your response for Problem 1, you will receive a copy of Problem 2 and a copy of Problem 3. Your team will then have the time remaining in the team round to complete a response for each problem.

Note: if your team completes Problem 1 before the end of the allotted time, you may submit it and receive copies of Problem 2 and Problem 3 in advance.

1. You may use the given rectangular solid to help you solve this problem.

   The dimensions of a rectangular solid are 2, 3 and 4 units. Determine the length of the shortest path that meets the following two conditions:

   (a) The path connects a pair of opposite vertices.
   (b) The path can be drawn on the surface of the solid.

2. Note that the set of **positive integers** is $\{1, 2, 3, 4, 5, \ldots\}$.

   Although $\sqrt{2} + \sqrt{3}$ does not equal the square root of a positive integer, $\sqrt{27} + \sqrt{48}$ does.

   (a) Find a positive integer n such that $\sqrt{27} + \sqrt{48} = \sqrt{n}$.
   You must provide written work to show why $\sqrt{27} + \sqrt{48} = \sqrt{n}$.

   (b) Find another example like the one in part (a).
   In other words, find positive integers a, b, and c such that $\sqrt{a} + \sqrt{b} = \sqrt{c}$. a, b, and c cannot be square numbers.
   You must provide written work to justify your answer.

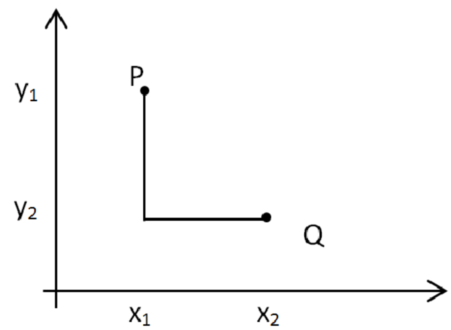
   (c) Find every set of positive integers a, b, and c such that

   - a, b, and c are all less than 35
   - $\sqrt{a} + \sqrt{b} = \sqrt{c}$
   - a, b, and c are distinct
   - None of $\sqrt{a}$, $\sqrt{b}$, and $\sqrt{c}$ is an integer.

3. For two points, $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ in a coordinate plane, the **Taxicab Distance** between P and Q is

$$d(P, Q) = |x_2 - x_1| + |y_2 - y_1|$$

Illustrated below, it measures the distance as a taxicab would travel on a rectangular grid of city streets.



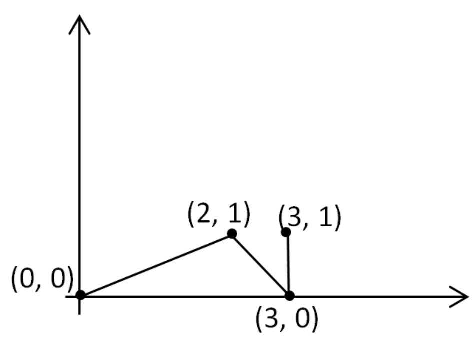
(a) Find $d(R, S)$ if $R = (1, 6)$ and $S = (4, 2)$.

A **Taxicab Loop** is the set of all points that have the same Taxicab Distance from a given point, referred to as the **Dispatch Point**. The Taxicab Distance from the Dispatch Point to any point on the Loop is known as the **Radius of the Taxicab Loop**.

(b) Sketch a Taxicab Loop with Dispatch Point at (0, 0) and with Radius 1.

The **Taxicab Length** of a path consisting of straight line segments is the total of the Taxicab Distances from one segment endpoint to the next on the path.

For instance, the Taxicab Length of the path illustrated below is 3 + 2 + 1 = 6.



(c) Can we use this to calculate the length of a Taxicab Loop? If not, explain why not. Otherwise, calculate the ratio of the length of a Taxicab Loop to the Radius of the Loop.