The Twenty-Ninth Annual
Eastern Shore High School Mathematics Competition

November 8, 2012

Team Contest Exam

Instructions

Answer as many questions as possible in the time provided. To receive full credit for a correct solution, show all work and provide a clearly written explanation. Solutions will be judged based on correctness, completeness and clarity. (Little credit, if any, will be given for a solution consisting of just a number or a single sentence.)

All work and answers must be written on the provided sheets of plain white paper. Use only one side of each sheet of paper, and start each new problem on a new sheet of paper. Write your team name (that is, the name of the school which you are representing) at the top of each sheet that you turn in for scoring.

At the start of the team round, your team will receive a copy of only Problem 1. Your team must submit a response to Problem 1 within the first 15 minutes of the team round time interval.

When you submit your response for Problem 1, you will receive a copy of Problem 2 and a copy of Problem 3. Your team will then have the time remaining in the team round to complete a response for each problem.
Note: if your team completes Problem 1 before the end of the allotted time, you may submit it and receive copies of Problem 2 and Problem 3 in advance.

1. In a recent mathematics department meeting at WLM University, the 33 members present ranked who they thought had invented calculus. The members were all required to choose exactly one first place candidate, one second place candidate and one third place candidate. The three candidates to be voted upon were Leibniz, Newton, and the famous Jerk (otherwise known as the third derivative of position with respect to time).

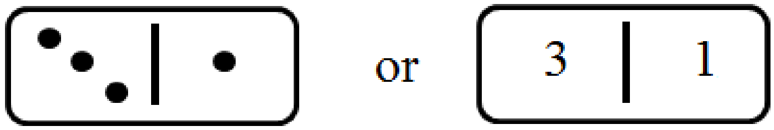
(a) There are n ways that each voter can rank the candidates. Find n.

(b) Each member is given 3 points to distribute to the candidates (2 points for first choice and 1 point for second choice). Find the number of points Newton, Leibniz, and the Jerk each receive if the votes were distributed amongst three of the rankings as shown in the following table:

Rank	19 voters	10 voters	4 voters
1st	L	J	N
2nd	N	N	J
3rd	J	L	L

(c) Is there a way to distribute the 33 votes such that L wins the most first place rankings BUT does not earn the most points? If there is, create a table (similar to the one above) for such a situation and explain how the table justifies your answer.

2. Consider a set of "double-N dominoes." Each domino has two regions separated by a bar. The dots which appear on dominoes are also called pips. Each region in a set of double-N dominoes contains one of the numbers in $\{0,1,\ldots,N\}$. A domino may be referred to by the numbers in the respective regions. The domino shown below is a 3-1 (or 1-3) domino.



Note: you have been given a set of double-3 dominoes.

- A double-0 set has 1 domino. i.e., the 0-0 domino.
- A double-1 set has 3 dominoes. i.e., the 0-0, 0-1 and 1-1 dominoes.
- A double-2 set has 6 dominoes. i.e., the 0-0, 0-1, 1-1, 0-2, 1-2 and 2-2 dominoes.

(a) How many dominoes are in a set of double-7 dominoes?

(b) Let N be a non-negative integer. How many dominoes are in a set of double-N dominoes? Express your answer in terms of N.

Another way to examine double-N dominoes is to examine the sum of the pips. For example:

- In a double-0 set the sum of the pips is 0. i.e.,(0+0)=0.
- In a double-1 set the sum of the pips is 3. i.e.,(0+0)+(0+1)+(1+1)=3.
- In a double-2 set the sum of the pips is 12. i.e.,(0+0)+(0+1)+(1+1)+(0+2)+(1+2)+(2+2)=12.

(c) What is the sum of the pips in a set of double-7 dominoes?

(d) Let N be a non-negative integer. What is the sum of the pips in a set of double-N dominoes? Express your answer in terms of N.

3. (a) Find a 3-digit number with the following properties:

   1. Each of the digits 1, 2, and 3 is used exactly once in the number.
   2. If we ignore all but the first n digits of the number, we obtain a number divisible by n.

   In other words, find a number abc so that the number abc is divisible by 3, the number ab is divisible by 2, and the number a is divisible by 1.

   If no such number exists, give an explanation for why this is the case.

   (b) Find a 4-digit number with the following properties:

   1. Each of the digits 1, 2, 3, and 4 is used exactly once in the number.
   2. If we ignore all but the first n digits of the number, we obtain a number divisible by n.

   In other words, find a number $abcd$ so that the number $abcd$ is divisible by 4, the number abc is divisible by 3, the number ab is divisible by 2, and the number a is divisible by 1.

   If no such number exists, give an explanation for why this is the case.

   (c) Find a 5-digit number with the following properties:

   1. Each of the digits 1, 2, 3, and 4, and 5 is used exactly once in the number.
   2. If we ignore all but the first n digits of the number, we obtain a number divisible by n.

   If no such number exists, give an explanation for why this is the case.

   (d) Find a 6-digit number with the following properties:

   1. Each of the digits 1, 2, 3, and 4, 5, and 6 is used exactly once in the number.
   2. If we ignore all but the first n digits of the number, we obtain a number divisible by n.

   If no such number exists give an explanation for why this is the case.